A New Model of Generalized Invertor and Its Applications

Jun Chen, Wei Hong, and Changhong Liang

***Abstract*— A new model of generalized invertor is presented. The model is generalized from the conventional K-, J-invertor. So it is easy to be applied directly to design microwave circuits such as impedance transformers, filters, and couplers of any complex discontinuities, when the symmetrical and asymmetrical networks are involved. The theory based on the new model is developed to enrich the microwave theory on network synthesis. Examples are given to show the effectiveness and convenience in application.**

I. Introduction

The microwave theory on network synthesis has been regarded as an additional theory for a long time because it depends heavily on both the theory of lumped parameters and the conception of equivalence. Therefore, the model of K-, J-invertor, depicted in Fig. 1, is of great importance to the independent development of the microwave theory on network synthesis with respect to such meaning. The ideal invertor based on coupled elements was presented by Cohn [1]. The characteristic of quarter-wave transformer was fully applied to the impedance invertor when the direct-coupled-resonator filter was analyzed by Cohn. He pointed out that the invertor can be characterized by the impedance transformation of transmission line of the quarter-wave length, $Z_1 = K^2/Z_2$ and $Y_1 = J^2/Y_2^2$, where K is the characteristic impedance of the K-invertor and J is the characteristic admittance of the J-invertor. Duality is easy to be found between the two kinds of invertors. Therefore, a shunt capacitance can be transformed to a series inductance, $L = K^2C$ and vice versa $C = J^2L$. Then Riblet [2], Young [3], and Mattahaei *et al.* [4] made a deep study and development of the K-, J-invertor, which is now one of the most important bases of the microwave theory on network synthesis. The concept of invertor is systematically employed to design microwave devices today.

But the theory of the K-, J-invertor still remains problems in practice. First, it cannot be directly applied to the networks of discontinuity, so it fails to be used in most general cases. Second, it cannot be used to the networks of asymmetry, so the equivalence of the invertor, losing generality, is of fewer circuits of lumped elements, thus making the application limited.

In view of this situation, starting from the most general network, we present a new model of generalized invertor that includes the symmetric and asymmetric cases. The new model with the theory developed expands the application and is easily applied to the design of microwave circuits, such as impedance transformers, various filters, and couplers.

II. New Model of Generalized Invertor

The new model of generalized invertor includes the symmetric- and asymmetric-type. It consists of a general lossless network $[A]$ and two ideal transmission lines (with electrical lengths θ, φ respectively) on sides of the network. As shown in Fig. 2 the A-parameter of the lossless network is given as

$$[A] = \begin{bmatrix} a_{11} & ja_{12} \\ ja_{21} & a_{22} \end{bmatrix} \tag{1}$$

where a_{11}, a_{12}, a_{21}, and a_{22} are reals demanded by the lossless condition.

The over-all A-parameter can be obtained by cascading the two transmission lines to the lossless network $[A]$, so there is (2), shown at the bottom of the next page.

The symmetric-type has the same definition as the K-invertor. The overall A-matrix of the such network in Fig. 2 is

$$[A_K] = \begin{bmatrix} 0 & \pm jK \\ \pm j\frac{1}{K} & 0 \end{bmatrix}. \tag{3}$$

The asymmetric-type is defined as the N-invertor. The overall A-matrix of the such network therefore is written as

$$[A_N] = \begin{bmatrix} \pm\frac{1}{N} & 0 \\ 0 & \pm N \end{bmatrix}. \tag{4}$$

As A-matrix is given by a certain discontinuity, solutions of K or N and θ, φ (here, θ, φ can be positive or negative) can be derived from simultaneous equations (2) and (3), as well as (2) and (4).

It is easy to derive the K-invertor of the symmetric-type. From (2) and (3) we have

$$\begin{cases} \theta = \frac{1}{2}\left[\tan^{-1}\left(\frac{a_{11}+a_{22}}{a_{21}+a_{12}}\right) + \tan^{-1}\left(\frac{a_{11}-a_{22}}{a_{21}-a_{12}}\right)\right] \\ \varphi = \frac{1}{2}\left[\tan^{-1}\left(\frac{a_{11}+a_{22}}{a_{21}+a_{12}}\right) - \tan^{-1}\left(\frac{a_{11}-a_{22}}{a_{21}-a_{12}}\right)\right] \end{cases} \tag{5}$$

and

$$K = \frac{1}{2}\left[\sqrt{(a_{11}+a_{22})^2 + (a_{21}+a_{12})^2} + \sqrt{(a_{11}-a_{22})^2 + (a_{21}-a_{12})^2}\right]. \tag{6}$$

If network $[A]$ is reciprocal, (6) is simplified in the form:

$$K = \frac{1}{2}\left(\sqrt{P^2+2} + \sqrt{P^2-2}\right) \tag{7}$$

where

$$P^2 = a_{11}^2 + a_{12}^2 + a_{21}^2 + a_{22}^2. \tag{8}$$

The N-invertor of asymmetric-type has the similar formulas from (2) and (4)

$$\begin{cases} \theta = \frac{1}{2}\left[\tan^{-1}\left(-\frac{a_{21}+a_{12}}{a_{11}+a_{22}}\right) + \tan^{-1}\left(-\frac{a_{21}-a_{12}}{a_{11}-a_{22}}\right)\right] \\ \varphi = \frac{1}{2}\left[\tan^{-1}\left(-\frac{a_{21}+a_{12}}{a_{11}+a_{22}}\right) - \tan^{-1}\left(-\frac{a_{21}-a_{12}}{a_{11}-a_{22}}\right)\right] \end{cases} \tag{9}$$

and

$$N = \frac{1}{2}\left[\sqrt{(a_{11}+a_{22})^2 + (a_{21}+a_{12})^2} - \sqrt{(a_{11}-a_{22})^2 + (a_{21}-a_{12})^2}\right]. \tag{10}$$

For the reciprocal network $[A]$, (10) may be simplified as

$$N = \frac{1}{2}(\sqrt{P^2+2} - \sqrt{P^2-2}). \tag{11}$$

Equations (6) and (10) determine the impedance ratio (IR) of the generalized invertor. The electrical lengths θ, φ are given by (5) and (9). Further study of the generalized invertor shows the interesting properties:

1) The IR is uniquely determined by A-parameter and has no relation with θ, φ. That means the IR is related with the discontinuity only.

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TABLE I
TRANSFORMATION OF NONNORMALIZED NETWORK OF TWO-SIDE CHARACTERISTIC IMPEDANCES

	for θ, φ	for K or N
a_{11}	$A_{11}Z_{02}$	$A_{11}/\sqrt{Z_{01}Z_{02}}$
a_{12}	A_{12}	$A_{12}\sqrt{Z_{01}Z_{02}}$
a_{21}	$A_{21}Z_{02}$	$A_{21}\sqrt{Z_{01}Z_{02}}$
a_{22}	$A_{22}Z_{01}$	$A_{22}\sqrt{Z_{01}/Z_{02}}$

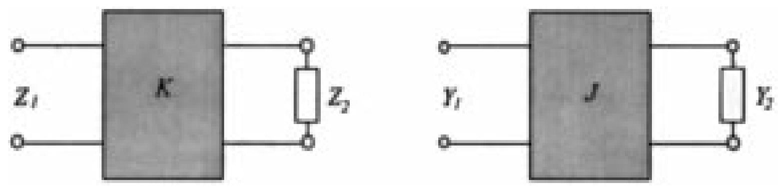


Fig. 1. The K- and J-invertor. $Z_1 = K^2/Z_2$, $Y_1 = J^2/Y_2^2$. The relation between the two characteristic impedances is $K = 1/J$.

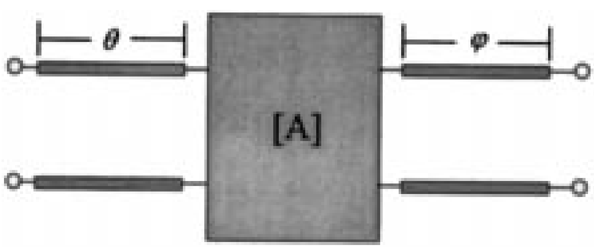


Fig. 2. New model of generalized invertor.

2) A real IR can be obtained through selection of a certain reference plane. So, θ, φ act as the factors to determine reference planes.
3) The relation between the electrical lengths of the symmetric-type θ_S, φ_S and those of the asymmetric-type θ_A, φ_A is easy to obtain

$$\varphi_A = \varphi_S \pm n\pi \quad \text{and} \quad \theta_A = \theta_S + \frac{\pi}{2} \pm n\pi \tag{12}$$

or

$$\varphi_A = \varphi_S \pm \frac{\pi}{2} \pm n\pi \quad \text{and} \quad \theta_A = \theta_S \pm n\pi. \tag{13}$$

Obviously, for the same A-parameter, the N-invertor can be transformed to the K-invertor so long as a quarter-wave transformer is cascaded to the right or left side of the invertor. So, it is the same with the K-invertor. This implies that the symmetrical network can be converted to the asymmetrical one and vice versa.
4) The relation between the K-invertor and the N-invertor should be presented. Comparison of (7) and (10) yields the following equation for the same A-parameter:

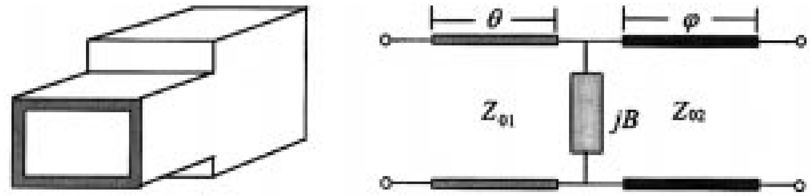
$$N = \frac{1}{K}. \tag{14}$$

5) Small changes are made for the formulas when the characteristic impedances of the two sides are different, namely Z_{01} (for θ) and Z_{02} (for φ). Table I shows the changes.
6) The formulas for the J-invertor (the admittance invertor) can be deduced from formulas of the K-invertor (the impedance invertor) in accordance with the duality principle. For the same reason, formulas for the generalized admittance invertor, so called M-invertor, can also be deduced from its dual N-invertor for the asymmetric-type.

III. EXAMPLES

Two examples of asymmetric networks are given in this section. It should be point out that the discontinuities in these cases are usually evaded in previous theories. The first example illustrates the effectiveness of the new model and the efficiency of the formulas. The second example shows the application of design by the formulas presented.

$$\begin{aligned}[A]_{\text{overall}} &= \begin{bmatrix}\cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}a_{11} & ja_{12} \\ ja_{21} & a_{22}\end{bmatrix}\begin{bmatrix}\cos\varphi & j\sin\varphi \\ j\sin\varphi & \cos\varphi\end{bmatrix} \\ &= \left[\begin{matrix} a_{11}\cos\theta\cos\varphi - a_{12}\cos\theta\sin\varphi - a_{21}\sin\theta\cos\varphi - a_{22}\sin\theta\sin\varphi \\ j(a_{11}\sin\theta\cos\varphi - a_{12}\sin\theta\sin\varphi + a_{21}\cos\theta\cos\varphi + a_{22}\cos\theta\sin\varphi)\end{matrix}\right. \\ &\qquad \left.\begin{matrix} j(a_{11}\cos\theta\sin\varphi + a_{12}\cos\theta\cos\varphi - a_{21}\sin\theta\sin\varphi + a_{22}\sin\theta\cos\varphi) \\ -a_{11}\sin\theta\sin\varphi - a_{12}\sin\theta\cos\varphi - a_{21}\cos\theta\sin\varphi + a_{22}\cos\theta\cos\varphi\end{matrix}\right]\end{aligned} \tag{2}$$



(a) (b)

Fig. 3. (a) Configuration of the waveguide stepped-impedance transformer and (b) its equivalent circuit at the discontinuity junction.

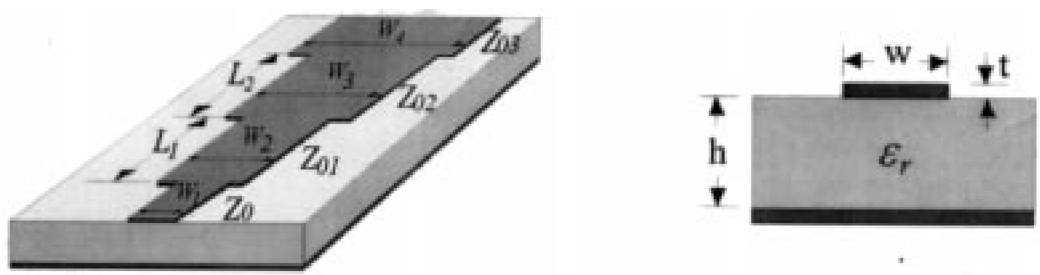


Fig. 4. Structure of the designed microstrip stepped-impedance transformer.

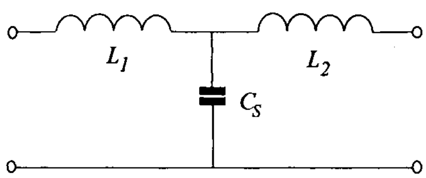


Fig. 5. Equivalent circuit of the stepped discontinuity of microstrip.

A. Waveguide Stepped-Impedance Transformer

The waveguide stepped impedance transformer is one of the most typical example [see Fig. 3(a)]. The characteristic impedances of the two sides are Z_{01} and Z_{02} (or Y_{01} and Y_{02}), respectively. The admittance of the stepped discontinuity is jB [Fig. 3 (b)]. So, the A-parameter of the discontinuity is

$$[A] = \begin{bmatrix} A_{11} & jA_{12} \\ jA_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jB & 1 \end{bmatrix}. \tag{15}$$

The IR of the asymmetric network is given by substituting the A-parameter into (11) with the help of Table I. We have

$$N = \frac{1}{2\sqrt{Z_{01}Z_{02}}}\left[\sqrt{(Z_{01}+Z_{02})^2 + B^2 Z_{01}^2 Z_{02}^2} - \sqrt{(Z_{01}-Z_{02})^2 + B^2 Z_{01}^2 Z_{02}^2}\right]. \tag{16}$$

Considering $B \to 0$ (neglecting the discontinuity) and $Z_{02} < Z_{01}$ (without losing generality), we rewrite (16) as

$$N = \sqrt{Z_{02}/Z_{01}} \tag{17}$$

which is the well-known formula of the impedance ratio of waveguide stepped-impedance transformer.

Taking account of the discontinuity, we have

$$\begin{cases} \theta = \frac{1}{2}\left[\tan^{-1}\left(\frac{B}{Y_{01}-Y_{02}}\right) + \tan^{-1}\left(\frac{B}{Y_{01}+Y_{02}}\right)\right] \\ \varphi = \frac{1}{2}\left[\tan^{-1}\left(\frac{B}{Y_{01}-Y_{02}}\right) - \tan^{-1}\left(\frac{B}{Y_{01}+Y_{02}}\right)\right] \end{cases}. \tag{18}$$

The same formulas can be obtained with difficulty by using the complicated concept of reference planes for high-low impedance if the discontinuity is considered with the ideal invertor model [4].

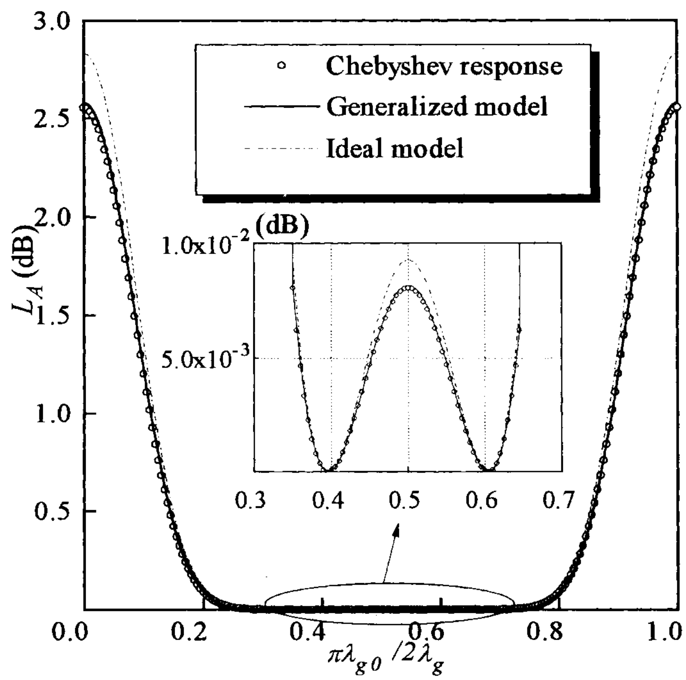


Fig. 6. Response curve of impedance transformer.

B. Design of Microstrip Stepped-Impedance Transformer

The structure of a typical two-section microstrip stepped-impedance transformer, designed at center frequency $f_0 = 5$ GHz, is depicted in Fig. 4. The characteristic impedances are Z_0, Z_{01}, Z_{02}, Z_{03} from the high to the low. The lengths L_1, L_2 of the two middle sections have their electrical lengths θ_1, θ_2. $W_i (i = 1, 2, 3, 4)$ are widths of the microstrip. The thickness of the substrate is $h = 1.0$ mm. The thickness of the strip is $t = 0.1$ mm. The relative permmitivity of the substrate is $\varepsilon_r = 9.60$. The relative bandwidth demanded is $W_q = 0.6$. The impedance ration is 5 : 1 (the input impedance $Z_0 = 50\ \Omega$ and the output impedance $Z_{03} = 10\ \Omega$). The Chebyshev prototype is adopted by the impedance transformer as the ratio for each section is 5.000 : 3.1765 : 1.5741 : 1.000. The equivalent circuit and expression for the discontinuity of the step is based on the work done by Benedek [5] (Fig. 5)

TABLE II
COMPARISON ON THE DESIGN OF THE IMPEDANCE TRANSFORMER BY THE IDEAL INVERTOR MODEL AND THE GENERALIZED INVERTOR MODEL

		Ideal design		Generalized design	
0		0.8595mm		0.8595mm.	
1		2.0790mm		2.0798mm	
2		5.6937mm		5.7006mm	
3		9.9254mm		9.9254mm	
L_1	θ_1	5.5676mm	0.2500π	5.6673mm	0.2545π
L_2	θ_2	5.5676mm	0.2500π	5.5676mm	0.2527π

Comparison of the ideal design and the generalized design for the impedance transformer is shown in Table II. The response curves of the two transformers are simulated and plotted in Fig. 6, where the ideal design (using the ideal invertor model) is unable to deal with the discontinuity of the step and thus evades it [3]. On the contrary, the generalized design (using the generalized model) copes with the discontinuity easily. The design comparison shows the formulae given by the generalized model is more accurate than the ideal one as discontinuities are considered.

IV. CONCLUSION

The new model of the generalized invertor is proposed for the symmetric- and asymmetric-type. It is suitable to general circuits networks instead of fewer lumped circuits. The new model is apt to be applied to the design of the passive network directly.

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A Design Procedure for Monolithic Matrix Amplifier

Claudio Paoloni and Stefano D'Agostino

***Abstract*—A procedure for the design of monolithic matrix amplifier is proposed. A simplified expression for small signal gain based on unilateral field-effect transistor (FET) model is derived. In particular, the Design-Oriented FET model previously published is adopted. The introduction of a set of design charts allows the designer a fast and accurate prediction of low frequency gain and 3-dB cutoff frequency of a given matrix amplifier. Good agreement with experimental data and simulations confirms the validity of the proposed design method.**

I. INTRODUCTION

Matrix amplifier, due to the combination of multiplicative and additive amplification principle, represents a suitable solution when very wide frequency band and high gain are required. Many realizations have been recently presented in the literature [1]–[3].

Theoretical studies on matrix amplifier were also published [4]. Unfortunately, the complexity of the mathematical formulations limits their applicability to practical design. This becomes especially true when the parasitics of the field-effect transistor (FET) composing the amplifier are included in the computation. Approximate expressions for small signal gain were given, but partially, help to solve the problem.

In this paper a simple formula to accurately predict the matrix amplifier frequency response will be proposed. The formula in conjunction with the Design-Oriented FET model reported in [5] provides an effective and accurate prediction of matrix amplifier performance. As it was demonstrated, the Design-Oriented FET model effectively takes into account the FET parasitics, maintaining at the same time the simplicity of the unilateral model.

A straightforward process-independent graphical design procedure for matrix amplifier is also defined to provide the designer a fast

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